

MAS125 Calculus II 2006-2007

Problem sheet for exercise class 5

- **Please attend the exercise class to which you have been assigned**
- The instructor will present the starred problems in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online after 5pm Friday.
- **The mid-term test will be on Thursday 22 February and taken on-line.**
- **There will be a revision lecture at 3pm Tuesday 20 February in Drapers.**

(*)Problem 1: Assuming that the series

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \dots$$

converges to $\tan x$ for $-\pi/2 < x < \pi/2$, find the first five terms of the series for $\ln |\sec x|$ and $\sec^2 x$. Indicate the values of x for which each series converges.

(*)Problem 2: Starting with the geometric series for $1/(1-x)$, show how one can operate on this to obtain the sum of the series

$$\sum_{n=1}^{\infty} (n^2/2^n) = \frac{1}{2} + 1 + \frac{9}{8} + 1 + \frac{25}{32} + \dots$$

Hint: the operations you need are differentiation, multiplication by x and finally selection of a value of x .

Problem 3: Show that the terms of the alternating harmonic series

$$S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$$

can be rearranged to give a new series which converges to $1/3$. If one multiplies the original series by two, show that it can be rearranged to look like S again. This seems to imply $S = 2S$, but the sum of a conditionally convergent series depends on the arrangement of the terms, so the values of S are different!

Problem 4: Determine which of these series converges absolutely or conditionally or diverges:

$$(a) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^3 + 1}, \quad (b) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}, \quad (c) \sum_{n=1}^{\infty} (-1)^{n+1} (\sqrt{n+1} - \sqrt{n}).$$

Problem 5: Find the values of x for which the following series converge absolutely or conditionally, specifying both the radius and interval of convergence:

$$(a) \sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n}, \quad (b) \sum_{n=0}^{\infty} \frac{x^n}{n\sqrt{n}3^n}, \quad (c) \sum_{n=1}^{\infty} n^n x^n.$$